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Constructing the Yield Curve for Sri Lankas Government Bond Market

DEWUNDARA LIYANAGE P. M. RATHNASINGHA[1]*;
KANGARA PATHIRANNEHELAGE N. S. DAYARATHNE [2]
[1,2] University of Colombo, Colombo, Sri Lanka

Abstract: The primary objective of the study was to construct the most applicable model that can be used for fitting yield curves for Sri Lanka Government Bonds. It is important to adhere to a scientific method of deciding the yields for respective securities in line with the yield curve than ad-hock and personal judgments for bond Investors, traders, and monetary authorities. Nelson-Siegel model used Ordinary Least Squares after fixing the Lambda parameters to make the model linear and flexible enough to handle. Primary examination of the Nelson-Siegel parsimonious approach on Sri Lanka Government Securities yields fit the yields having an average R^2 of 91.4%. But R^2 was not consistent throughout the entire study period i.e., 2010 to 2018 as R^2 fell below 80% in some of the periods. Hence, the different discounting function was used to enhance the accuracy and predictability of the model applying Ordinary Least Squares technique. Two models were derived for the final test and evaluation and found one model with two exponential decay functions and tenor as another predictor. This model exhibits over 97% R^2 even after fixed for two Lambdas. Further, the out-of-sample predictability of the model surpassed the other models with an average of higher R^2 values and less Mean Residual Sum of Squared values. Clustering algorithms (listed under Weka) were used to examine the different patterns of the yield curve during the study period. The same model was re-constructed for identified clusters fixing separate Lambda but the comparison of separate lambda for each cluster and overall lambda did not indicate significant differences. The final model comprises the traditional Nelson-Siegel model, but more towards the Dobbie-Wilkie Model, which accounted for two exponential dece functions and a time variable. In Sri Lanka, very few attempts have been made to understand the behavior of the yield curve. That resulted to have controversial and unorthodox practices to judge the optimal yields for respective government bonds, especially for the longer tenor bonds beyond 5 years. Monetary authorities are geared to develop and extend the Sri Lankan Government securities Bond yield curve beyond the 10 years (tenor) which will encourage foreign investors to invest in Sri Lanka bonds. Countries with active longer tenor yield curves have been benefited from FDI.

Keywords: Yield curve, Nelson-Siegel model, Government bond market, Sri Lanka

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INTRODUCTION

One of the most challenging problems faced by capital markets across the world is forecasting the future trend of interest rates. From an economic standpoint, it has been described as forecasting the term structure of interest rates and from the mathematicians perspective, it is considered as modeling the yield curve. Understanding the relative movement of the yield curve of a country will not only important to stakeholders in the capital market but is indirectly important for the public. Because salient features of the yield curve are attached to the economic principle of society. It can be used to forecast the growth, recessions, and even exchange rate between currencies.

The yield curve is the relationship of yields and time to maturity of different securities (Rubaszek, 2016; Wang & Chen, 2020) on a given particular day. Therefore, many different yield curves can be observed for different dates. Hence, this can be considered as three-dimension or penal data, where yield, the maturity of the securities, and time would be the three dimensions (Christensen, Lopez, & Rudebusch, 2010). Every debt instrument (Deposits, government securities, corporate bonds, commercial papers, etc.) has its yield curve. The Bond market of a country consists of many securities (financial debt instruments). The bonds are broadly divided into corporate bonds and government bonds based on the issuer of the

*Corresponding author: Dewundara Liyanage P. P. Rathnasingha
†Email: prasath@dfn.cmb.ac.lk

securities. As the corporate bond market is not matured enough to extract the required data, it was not considered for this study. In contrast to the corporate bond market, the government bond market in Sri Lanka is very much active, and required data are available.

The government Bond market is known to be the foundation for deriving yields of other financial instruments, as the capital marketers believe that a Treasury bond is a risk-free investment instrument, with regard to credit risk. Therefore, studying the yield curve of the government bond market provide basic insights into the term structure of interest rates, of a country (Mineo, Alencar, Moura, & Fabris, 2020). The shape of the yield curve (upward, downward, flat) is an important indicator for understanding the behavior of short-term interest rates, as well as long-term interest rates of a country. The traditional way of illustrating the yield curve is to draw a scatter plot, where the y-axis being the yield and the x-axis being the tenors or maturity of the securities. Essentially, the line that connects the plots is in a curve nature.

Regression-type of techniques were used by McEnally (1987) to develop two models; Bradley-Crane model & Elliot-Echols. Underline principle of this technique is to estimate the function parameters by minimizing the squared differences between the actual and fitted yields to maturity. Dobbie-Wilkie model by Dobbie and Wilkie (1977), Super-Bell model by Bolder and Streliski (1999) are some of attempted to model the yield curve using this technique.

Empirical yield curve models tried to overcome many drawbacks that prevailed in the previous technique. Applying an appropriate mathematical discount function, thereby estimate the parameters, is the key principle behind the empirical models. Polynomial models (McEnally, 1987; Coleman, Fisher, & Ibbotson, 1992) are the most basic empirical yield curve models. One interesting development was to use piece-wise cubic polynomial which is known to be Spline-based methods models to improve the better fit of the curve to the data. One of the most accurate models used by many monetary authorities and scholars was the Parsimonious model developed by Nelson and Siegel (1987) and it was further improved by Svensson (1994). This model was able to capture the schematized features that describe the behavior of the forward rate curve.

Equilibrium models were based on theories about the nature of the stochastic process and proposed to model the yield curve based on such a process. Vasicek (1977) used Itos lemma process, Cox, Ingersoll Jr, and Ross (1985) propose the following risk-neutral process, Duffie and Kan (1996) manage to come up with the conditions on the drift, diffusion, and risk premium functions. As per the comparative studies, there is a continuous trade-off between smoothness and the fit factor in yield curve estimates, depends on which model that one uses (Nymand-Andersen, 2018).

Micro factor-based modeling is the other type of yield curve modeling that incorporates microeconomic factors as additional variables (Baskot, Orsag, & Mikerevic, 2018). It has been a common question of the ways that microeconomic variables impact yield curve behavior. Diebold, Rudebusch, and Aruoba (2006) incorporate macroeconomic factors into the yield curve models by using manufacturing capacity utilization, the federal funds rate, and annual price inflation. Dewachtera and Lyrioa (2002) studied the association of interest rate with the output gap, inflation, inflation expectation, and real interest rate. Still, those models are finding it difficult to provide a better fit to the entire yield curve and only successful in interpreting the behavior of the shot end of the yield curve. Hence, it has been found the most of the movements of the yield curve are still accounted for by the unobservable factors.

Debt market operations in Sri Lanka are can be categorized into two major sections and it is active. One is the interbank call money market and the other one is the Treasury bill/Bond (primary and secondary) market. But insignificant contribution can be seen from the coporate debts market on the basis of trading & issuing volume in the domestic market compared to the Treasury securities market. At the beginning of the government bond market in 1997, the maturities of Treasury bonds were limited only up to two years. It was used to test the markets appetite of trading government securities first time in Sri Lanka. With the impressive feedback, the maturities of securities were extended up to six years. Later in 2003, the government was able to extend the maturity to 20 years and it was further extended for 30 years in 2013.

The traditional treasury bills maturing three months, six months and twelve months are considered as short term securities and it represents the shooter end of the yield curve. Treasury bonds that are issued to be matured in two years to thirty years consider the medium term and long term securities and those represent the respective segments in the yield curve. The Treasury Bonds that have been issued in Sri Lanka comprised of two coupons to be paid during a particular calendar year. The coupon rate is been decide by the Central Bank of Sri Lanka (CBSL) from time to time. Currently, the Public Debt of Department on behalf of the CBSL has issued 60 different securities to the capital market. Some of the government securities are actively trading and some are not being trading as frequently as such. According to the treasury bill series available for investors published by CBSL, the longest maturity being issued is expected to mature on 1st March 2045, which was initially issued on 1st March 2015.

Research problem

Understanding the behavior of the interest rate (or yields of financial instruments) of a country is a difficult task under volatile economic conditions. The yield of short-term securities responds differently to the longer-term securities. Without having an idea about the behavior of the term structure of interest rate/yield curve, it is impossible to forecast future investment for a commercial entity. Further, it will be a tedious task for monetary authorities (CBSL) to raise funds for the national budget of a government without having an idea about the movement of interest rates in the country.

Research objectives

The main objective of this study is to develop a yield curve model for the government bond market in Sri Lanka. The other specific objectives are as follows:

- Examine a different kind of yield curve models.
- Identify the historical patterns of the yield curve in different economic time frames.
- Constructing the yield curve for the Sri Lankan government bond market using the statistical/mathematical tool and AI tools.
- Evaluate the method that is being used to construct the yield curve.
- Forecast the yield curve features for a period of six months.

Research questions

The research aims to answer the following research question:

- How will Sri Lanka Government bond yield behave in the future based on salient features of the yield curve?

LITERATURE REVIEW

Border sense, the yield curve can be depicted as a graph that illustrates the relationship between bond yields and respective maturities (remaining time for each security to deliver its principal obligation). This has become an important tool in fixed-income investing. Many investors across the global capital market use yield curve as a reference point/benchmark for forecasting interest rates, pricing bonds, and other financial derivatives. Developing and creating investment strategies for boosting total returns of the portfolio also depend on the yield curve. It has also become a reliable famous indicator to measure the economic activity of a given country.

The bond yield curve is a line graph that linked many scattered yield points across different maturities of different securities. It reflects the relationship between yields to maturity and time to maturity of bonds. Since we focus on the government bond yield curve, the asset class and credit quality will be the same for government bonds. The history of the Sri Lanka bond market goes way back to 1923 where Ceylon issue treasury loans under Colonial Treasury Bill Ordinance by Crown Agents in the U.K. In 1941, the country starts issuing short-term securities known as Treasury Bills where the principle maturity falling after three months, six months, and twelve months. These securities do not pay intermediately coupons compared to the common features of Treasury bonds. In 1997, Sri Lanka starts issuing a Treasury bond which is been attached a Coupon paid twice a year. The value of such coupon is fixed and the value is been decided by issuing party, which is the CBSL. Since then, the Sri Lanka Bond market has extended its maturity date of the bonds (extending the yield curve) by developing infrastructure as well as knowledge of the market participants. In 2003, the country issued bonds for up to 10,15, and 20 years. After ten years later the country issued security with the longest maturity period of 30 years.

The public Debt Department is the agent who issues these securities on behalf of the CBSL and in turn on behalf of the Government of Sri Lanka. CBSL has appointed a number of the agent to distribute government securities among the investors and they are known as Primary Dealers. In the primary market, the CBSL issue these government securities to Primary Dealers, and all the other investors will have to purchase those securities from Primary Dealers. This market is known as the secondary market. Buying and selling of government securities, commonly knowns as trading of securities are been done in this secondary market.

The movements of yields of different securities tend to behave differently. It can be very clear all the different securities have moved together in the long run but not in the short run. There are several basic yield curve shapes can be observed crossed many different capital markets (Choudhry, 2018; Oladunni, 2015). Normal or conventional shapes indicate yields are at average levels and the curve slopes gently upwards as maturity increases. The second common shape is the upward sloping or positive or rising scenarios, where the yields are at historically low levels while long rates (rates that are corresponding to longer maturities) substantially greater than short rates (rates that are corresponding to shorter maturities). The third basic shape has been identified as downward-sloping or inverted or negative shape, under this type of the yield levels are very high as per the historical data, but long-term yields (rates that are corresponding to longer maturities) are significantly lower than short rates (rates that are corresponding to shorter maturities). The fourth type of shape of the yield curve has been identified as humped, where yields are high with the curve rising to a peak in the medium-term maturity of the plot, thereafter it sloping downwards at longer maturities. These four basic curves are expected to exhibit in the different time frames of the economy due to many reasons.

Three different forms of yield are traditionally discussed in the literature as well as the capital market. Par Curve, Spot curve, and forward curve are the said different forms of yield curves (Melik-Parsadanyan, 2016). These have a significant association. Because Par Curve and Forward curve has been the derivative of the Spot curve. The value of a bond (Price) is been derived by the following equation;

$$pv = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \cdots \ldots + \frac{C+R}{(1+r_m)^m}$$

Where C - Fixed coupon value; r interest rate, R- Principle amount (Face value), m period of the bond maturity. This illustrates, the bond price been derived based on discounting the future cash flow using an appropriate discounting factor. That is $1/(1+r_i)m_i$, where $m_{(i)}$ is the time to maturity for the ith cash flow and $r_{(i)}$ is the associated spot interest rate.

In general yield curve has a common pattern. Such nature is been explained by three main theories; namely Pure expectation Theory, Market Segmentation Hypothesis, and Liquidity Premium Theory. These three theories explain the different features of the yield curve. The expected theory suggests that bond investors do not prefer bonds of one maturity over another, so they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity (Choudhry, 2018). On the other hand, it tells rates of the long-term bonds will equal to the average of short-term bonds. Therefore, the bonds of different maturities can be perfectly substituted. This indicates the return of two years investment horizon is equal for an investor who invests one year bond and at maturity he/she invest in another bond that is matured after one year period and an investor who invests two-year bond directly.

The market segmentation theory says the demand for securities that mature in different years (maturity of the securities) will depend on the investors perception (Oladunni, 2015). Therefore, demand is segmented along the yield curve. Due to this reason securities with different maturities cannot be perfectly substituted, because different investors have different preferences on one security over other security. Therefore, the different yield curves have different patterns based on supply and demand. It further illustrates, not only the investor preference but all so economic factors, e.g., Inflation may have a higher correlation with bonds that having longer maturity dates while excess liquidity may have an influence on bonds that having shorter maturity dates.

Every bond (even any security issued by the government) has a risk because it exposes to inflation, future interest rate expectation. Liquidity premium theory views bonds with different maturities can be a substitute but not perfect. Investors prefer short tenor bonds to long tenor bonds because shorter tenor bonds mature early than longer tenor hence, the risk such as inflation will be less on short tenor bonds. In order to substitute longer-tenor bonds over the shorter tenor bond, a positive premium needs to be paid to compensate the said risk for the longer tenor bond.

The yield curve is the relationship between yield and time to maturity. The early stage of the yield curve modeling was a focus on linking the isolated plots in the Scatter Plot via a linear function. Then it became a function that relates yield and time using traditional regression techniques. Bradley-Crane model which was formulated by McEnally (1987) was in the form as follows;

$$\ln(1 + r_i) = \beta_0 + \beta_1 t_i + \beta_2 \ln(t_i)$$

Where r_i - Yield to maturity of Bond i, t_i - term to maturity of bond I (in years), β - regression parameters.

Most of these regression-type models have one key drawback, they do not show an adequate fit of the data, because of their over-simplified presentations of the term structure of market rates (Šedivá & Marek, 2015). It is mainly because the actual or natural yield curve is smooth and curvature, but those models are weak on capturing this feature and not flexible enough to change according to the changing economic environment.

Usage of spline-based models allows rectifying some of the weaknesses of the above pure regression-based models. It uses a piece-wise regression which allows separate regression models for part of the curve and separate estimates for another part of the curve. These segmented points are denoted as Knot points. Cubic polynomial (McEnally, 1987), Exponential polynomial (Nelson & Siegel, 1987), gamma, Weibull, Gompertz, and lognormal functions based polynomial (Cairns, 1998) are some of the developments found in the literature.

Out of the above, the most discussed and used model to fit the yield curve is the model constructed by Nelson-Siegel. The most interesting feature of this model has been the simplicity and ability to capture the curve nature of the natural yield curve. The forward yield formula is been deified as follows;

$$f_t(\tau) = \beta_{1t} + \beta_{2t} e^{-\lambda_t \tau} + \beta_{3t} \lambda_t e^{-\lambda_t \tau}$$

Where, the βs & parameters,

The other important fact about this model is, it can be view as a simple mathematical model (constant plus a Laguerre function) as well as economical interpretation. There have been several attempts to increase the predictability power of the Nelson and Siegel model, Svensson 1994 increases the number of parameters by adding two extra parameters to allow for a capturing second hump in the forward rate curve. And many studies have been carried out (Šedivá & Marek, 2015; Choudhry, 2018; Mineo et al., 2020; Oladunni, 2015; Suimon, Sakaji, Izumi, & Matsushima, 2020) to test and enhance the based model and customized it to the respective countries.

In contrast to the above models, some researchers believe the interest rate (spot rate) follows a diffusion process or some sort of the nature of the stochastic process. They assumed the price of a discount bond depends only on the spot rate and the market is efficient. There are several such models (Holston, Laubach, & Williams, 2017; Yasir et al., 2020) that have been developed and those were named equilibrium models. Vasicek (1977) one of the founders of these type of model explains the levels of yields as a Wiener process.

$$dx = a(x,t)dt + b(x,t)dZ$$

Where dZ is a Wiener process and a and b are functions of x and t.

So he used the above Ito lema process and defined the yield model as follows;

$$dr = \alpha(\gamma - r)dt + \sigma dZ$$

Where,
r = the risk-free rate,
y = the expected long-term rate
α = the mean reversion rate
σ = the volatility of the rate
dZ = a normally distributed stochastic term.

But these models were not popular among researchers as well as academies because of the weak model fitting and inflexibility of handling these models. (Bauer & Rudebusch, 2020) suggest using microeconomic factors to predict future interest rates. De Pooter, Ravazzolo, and van Dijk (2010) found three key latent factors under exploratory factor analysis, the first principal component was found to be a real activity, the second component was express as the price dynamics and the third one was associated with monetary variables.

All those models are statistical models based on various mathematical functions. However, some scholars argue those models have failed to beat a simple random walk. It forced to explore the possibility of applying artificial intelligence tool to model the yield curve. They propose to use nonlinear, nonparametric models, machine learning techniques such as Support Vector Machines (SVM) and Artificial Neural Networks (ANN), etc. These are used in financial modeling to deal with non-linear problems in which the underlying dynamics and processes of the assets are unknown.

In Sri Lanka context, It has been very little academic research has been conducted on developing a yield curve for the bond market (Aazim, 2010; Karunasena, 2009). These studies are more into the behavior of Sri Lanka yield curve with respect to the monetary policy changes.

RESEARCH METHODOLOGY

Sri Lanka bond market

Debt market operations in Sri Lanka is two folds and both markets are very active. One is the interbank call money market and the other one is the Treasury bill/Bond (primary and secondary) market. The commercial papers debentures and unit trust are also falling to the money market in border sense. But not significant on the basis of trading and issuing volume in the domestic market compared to the Treasury securities market. The traditional treasury bills maturing three months, six months and twelve months are considered as short term securities and it represents the shooter end of the yield curve. Treasury bonds that are issued to be matured in two years to thirty years consider the medium term and long term securities and those represent the respective segments in the yield curve. The Treasury Bonds that have been issued in Sri Lanka are comprised of two coupons to be paid during a particular calendar year.

Methodology and tools

Understanding the term structure of interest rate would be the same as understanding the yield curve. Estimating the parameters of the yield curve using the Nelson-Siegel model will provide an insight into the different patterns of the yield curve. These parameters are time-dependent. ARIMA be used to model the future parameter of the model, thereby identifying the future yield curve. Clustering used to examine the different types of yield curve patterns. Neural Network will use to testify the accuracy of the parameters of the model.

This is a longitudinal study that entails collecting yields of eleven different securities, on a daily basis for the past eight years. The tenor of each security change on a daily basis tends to create a ruffle on the yield calculation. In order to eliminate such issues, either the yield calculated for standard tenors such as 1, 2, 3, etc. years by using extrapolation or the exact tenor 1.2, 2.6, etc. considered. The observed yields are referred to as Yield to Maturity (YTM- includes coupon effect). Since the Nelson-Siegel functional model deals with zero-coupon bond yields (spot yields), Excel function and bootstrapping used to convert the observed YTM to spot yields. Seventy percent (70%) of the data used as a training data set and the balance used as a test data set.

In order derive the parameters of the Nelson-Siegel Model (i.e., β1t, β2t, β3t, and λ) or any other model, the Ordinary Least Squared (OLS) method used to construct said three parameters after fixing the λ which is with the smallest error term. Three parameters selected minimizing the Residual Summers Squared (RMS) for each daily yield curve and the ARIMA modelused to forecast three parameters for one year forward time period. In order to derive the ARIMA model, SPSS/MINITAB used. Weka used as a data mining tool as an alternative method to construct a yield curve.

Population and time horizon

Sri Lanka has been experiencing a civil war that ended in May 2009. As a result, the behavior of many economic variables was distorted and was volatile due to uncertain prevailed at that period. To avoid the impact of such a nature, it was decided to extract data from 2010 onward. The data from 2010 to 2017 is expected to use as trading data set and the yield date of 2018 would be used as a testing data set.

Selection of the tenor of the securities

CBSL has been issuing government bonds for many different maturities as 91 days, 182 Days, and 364 Days are the standard short-term securities know and Treasury Bills. Longer tenor maturities ranged from 2 years to 30 years. But based on data availability it was decided to obtain yields of 91 days, 182 Days, and 364 Days, 2 years, 3 years, 4 years, 5 years, 6 years, 8 years, 10 years, and 20 years. The liquidity of bonds also plays a major role in deriving better yield curve, otherwise, the continuation of the data a challenge and also abnormal yields could be seen as illiquid bonds tend to showcase contradictory yields or higher dispersion of yields from the smooth yield curve.

It is not possible to get the yield for the exact tenor e.g., 2, 3, etc. as bonds that are trading on the secondary market are not in exact tenors. CBSL will issue bonds in exact tenor only in primary issues (But not always). Therefore, the discounted yield data that is been extracted from the secondary market need to readjust to reflect the exact tenor (e.g., 2 years, 3 years, etc.). Hence, it may require to apply interpolation and extrapolation to derive yields of the exact maturity. Since yield behaves in a curve as nature, it is required to use polynomial interpolation and extrapolation. The zero-coupon (or spot) yield curve is considered a true yield curve since it eliminates the effect of coupons. Most of the bonds issued in the globe have a coupon (annual or biannual) attached to it. All the bonds issued by the CBSL includes a biannual coupon. Therefore, Sri Lanka does not have a zero-coupon bond curve. Hence, it is required to derive zero-coupon yields from existing discounting yields. The bootstrapping method was used to derive the zero-coupon yields assuming the price of a secondary market bond is the same as pool (portfolio) of zero-coupon bonds of the same tenor.

Data frequency

It is important to have the most granularity level of data to allow a higher frequency (daily, weekly, monthly) of the data in the study. Hence, it was decided to extract daily discounted yields. This will allow reducing the frequency to weekly or monthly if required for any additional analysis with other economic variables.

Removal of outliers

Due to many reasons (bond liquidity, market excess money, etc.) the yields of a few bonds may still deviate significantly from the rest of the bonds in the yield curve. In order to eliminate such noise in the process of deriving yield curve estimation, such deviation should be identified as outliers and require to remove those from the sample. Bonds that having yields that deviate by more than two standard deviations from the average in each maturity bracket (in the case of using more than one bond to derive the yield of a particular tenor) treated as outliers (Andersen 2018).

DATA ANALYSIS

The type of data that was derived from CBSL data sheets, the preparation/pre-processing of those data in order to use in analysis, features and patterns of the data historical data and basic statistics was discussed in the descriptive data analysis. Further Weka was used to cluster the data in order to identify the key pattern of the yield curves during the studied period. Expected maximization and Kmean algorithm were used to select those clusters. Those were visualized to enhance the knowledge of the different pattern of the yield curves that was observable during the studied period

Since the yield and the remaining maturity of the security behave in a non-linear or curve nature we can not use one interpolation method to estimate/derive the yields for discrete tenors of the maturity. Hence, piecewise interpolation (different interpolation methods for different regions of the yield curve) was employed. The simplest method of interpolating between two points is by simply connecting them through a straight line. The function value between these points was estimated by simply using basic statistics. The cubic spline interpolation has been found as most appropriate for yield data interpolation (Du Preez, 2012; Williams et al., 2016).

The Microsoft Excel add-ins from SRS1-cubic splines 2.51 version was used to calculate the linear interpolation (linar_interp(tenor of existing securities, yields of existing securities, required tenor of the security)) and cubic spline interpolation (cubic_spline(tenor of existing securities, yields of existing securities, required tenor of the security). there are different shapes of yield curves for the selected period, it was decided to examine the similarities of the yield curves for this period. An unsupervised clustering technique in data mining was used to separate the yield curve pattern. Weka (Waikato Environment for Knowledge Analysis) was used to cluster the instances. Usually, weka provides possible clusters based on the algorithm we select (K-means, EM expectation maximization, etc.). As a rule of thumb, the number of clusters k can be calculated as follows; $k = (n/2)^{0.5}$ where n = number of data points. Therefore, the possible number of clusters were found as 7 (seven) for this study.

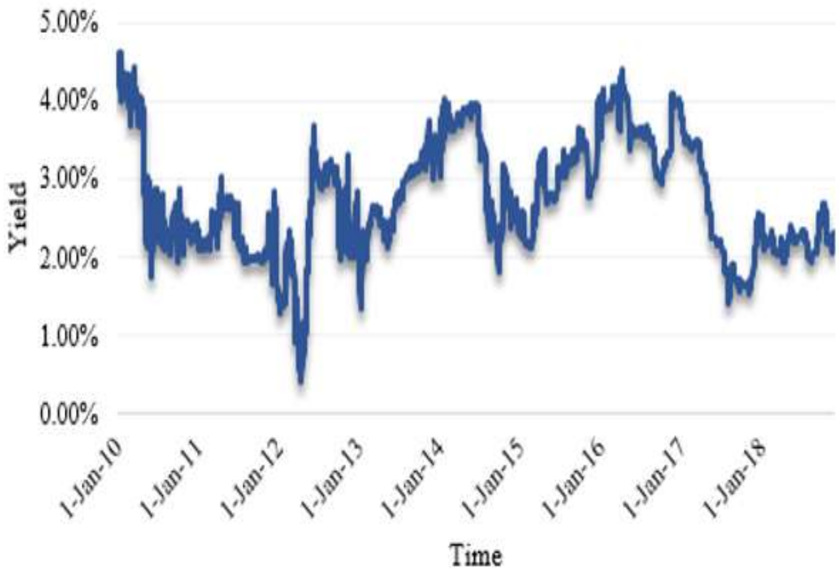

Figure 1. Yield spread of 10-year and 3-month securities (long term)

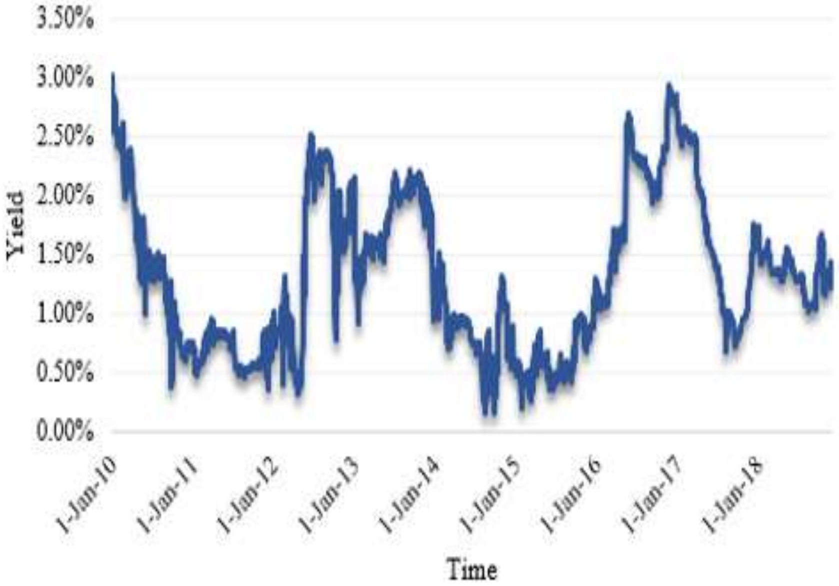

Figure 2. Yield spread of 2-year and 3-month securities (medium term)

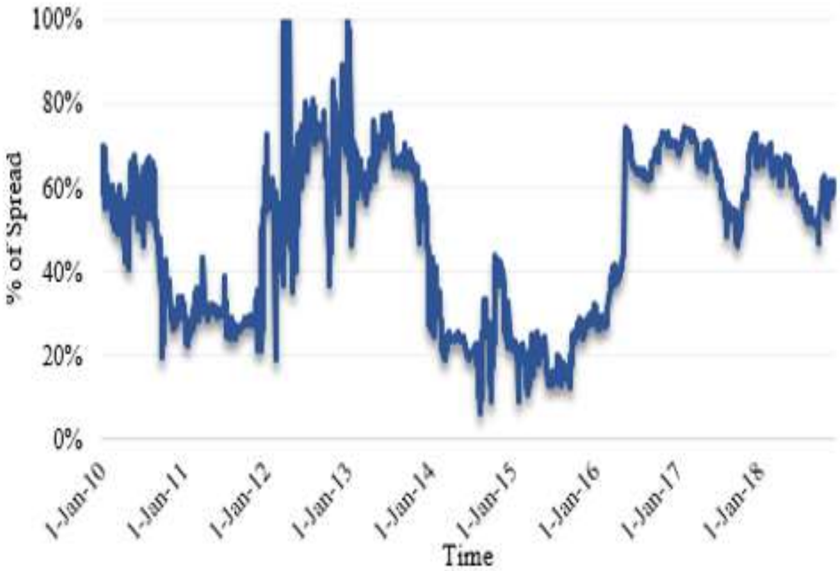

Figure 3. Percentage of medium-term yield spread against long-term yield spread

It can be observed that medium-term spread (2Y - 3M) has been less than 50% during the period of 2011 to 2012 and 2014 to 2016, which enable the yield curve to behave more steepen as the 10Y to 2Y yield spread is significantly higher. On the other hand, the yield curve has tended to be flatter than other periods. Interestingly the longer-term spread has been 3% for the period (end 2017 to end 2018). A similar scenario can be seen for an extended period only during mid-2010 to the end of 2011.

The steepening of the yield curve occurs when investors demand high-risk premiums to lend on long-term maturing securities to compensate for many adverse social and economic factors such as high inflation, budgetary and fiscal measurements which lead to the tight monetary situation than the current level yields. The steepening of the yield curve is associated with expectations of a rise in forwarding inflation or a deterioration in the fiscal position of the country (Aazim, 2010).

The flattening of the yield curve corresponds to a scenario in which long-term yields decline and approach the level of short-term yields. This situation can occur due to many reasons such as a change of investor expectations about the level of long-term bond yields, a possibility of declining future inflation, better performance of fiscal policies.

Simple linear interpolation was not considered as it is very clear that the scatter plots of yield data cannot be estimated using the linear line. Hence, Polynomial interpolation was used initially to understand the non-linear behavior of the yield curve. Further, it was a worthwhile attempt to identify the importance of the structures/forms of the dependent variable (t) that important to include in selecting the best model in the model selection.

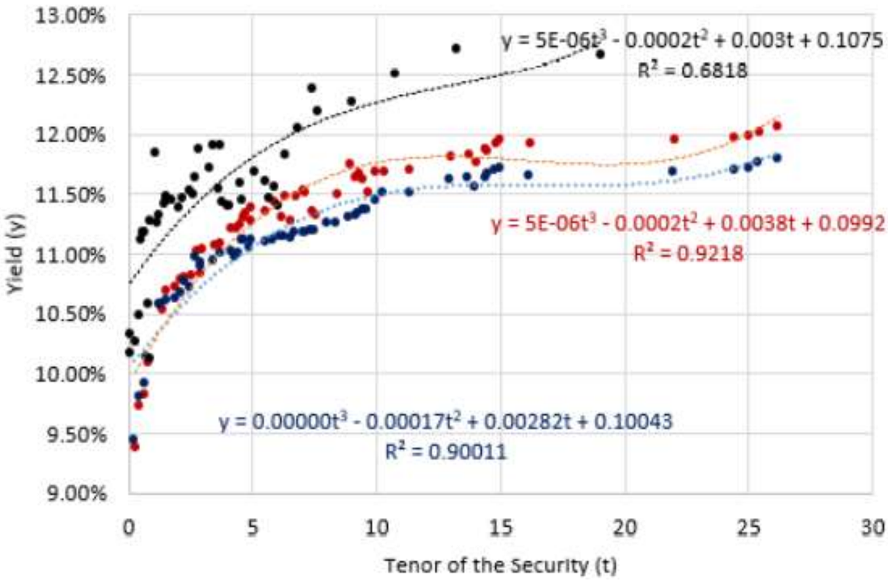

Figure 4. Yield curve with polynomial curve fitting

As pointed out earlier, Nelson and Siegel (1987) functional form was used as the base model for this study as it was considered the most simple and best-adapted model by many monetary authorities. The Ordinary Least Squared method was used to optimize the parameters $\beta0$, $\beta1$, $\beta2$, and λ.

NS Model was constructed for each day, and $\beta0$, $\beta1$, $\beta2$ and λ was estimated using the ordinary least squared method. The R^2 was also calculated for each regression model. The functional form of the model was the same, but the respective parameters were different each day. The model parameters were re-estimated after fixing λ as well. The solver was used to estimating the parameters after minimizing the overall residual sum of squared (residual sum of the square for each day) for the entire period (i.e., from 2010 2018).

Grid search of models

Since the standard NS model was unable to provide a better estimation of the yield curve for the entire period, it was decided to reinvent the suitable model for Sri Lanka government bonds. As the first step, the decay function related to said 1 was removed from the model and re-run the regression algorithm under the OLS method for each day and estimate the parameters for the two scenarios (with and without fixing the λ).

$$Y_t(\tau) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\lambda}\right) - - - \text{(Model: } MABM)$$

Similar to the NS model, the Model: MABM was also exhibited a poor fit to the actual data during the same period.

$$Y_t(\tau) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\lambda}\right) + \beta_2\tau - - - \text{(Model : } EABM)$$

During the polynomial interpolation, it was observed that the tenor of the security was significant and it has a higher coefficient value compared to other coefficient values. Hence, it was decided to include the to the model.

The model performs well when the λ parameters are not fixed. Fixing λ becomes important when the model is used for forecasting purposes. Hence, it was decided to add more decay function as follows and re-run the linear regression algorithm by applying OLS method to extract the relevant value for parameters.

$$Y_t(\tau) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\lambda_1}\right) + \beta_2\tau + \beta_3 \exp\left(-\frac{\tau}{\lambda_2}\right) - - - (\text{Model} : EDWM)$$

The Model: EDWM seems to have improved the R^2 for the entire period that was studied with and without fixing s. It performs well even after fixing for both the lambda. Since the Model: EDWM consists of six parameters, it was decided to remove the variable λ and re-run the parameter estimation process.

$$Y_t(\tau) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\lambda_1}\right) + \beta_2 \exp\left(-\frac{\tau}{\lambda_2}\right) - - - (\text{Model} : DWM)$$

But the overall result of Model: DWM was unable to overrule the performance of Model: EDWM. Removing λ did not improve the model by a greater extent. Hence, it was decided to limit the grid search of the suitable model by accepting Model: EDWM as the best and optimal model to capture the variation of the Sri Lanka Government bond yield curve.

The Model: EDWM was selected as the best model to use for creating a yield curve for Sri Lanka Government Securities. The final objective of this study was to forecast the yield curve based on the derived model.

Deriving the above model, the yield curve features were reduced to six as the Model: EDWM contains only six features or predictors. Out of which two parameters related to Lambda is fixed or constant. Hence, future yield curve can be forecasting by estimating future values for four Betas. In order to forecast values for $\beta 0$, $\beta 1$, $\beta 2$, $\beta 3$ time series analysis was performed. ARIMA models are the most frequently use algorithms to forecast future values assuming future values are dependent on past values through trends and seasonalities.

The expert model option available in SPSS was used to select the best model out of a different kind of ARIMA and exponential smoothing models. The same options are available under IBM Watson since SPSS was also a product under IBM. Model variables are transformed where appropriate using differencing and/or square root or natural log the transformation under expert modeler.

FINDINGS

Different types of model diagnostic tools were used to select the best model out of a series of models that were applied to predict the daily yield curves. Mean Squared Errors (MSE) was used to examine the goodness of fit for each model. Model: NS was the base model. Model: EDWM & Model: DWM was considered as optimal models for further consideration.

Table 1: Summary of the goodness of fit of the models

Model	Mean RSS	Mean R^2	Std. of R^2	Coefficient of Varience	No. of Parameters
NS	0.0000446	92.80%	0.086859	0.09355	4
NS (fixed λ)	0.0000938	91.40%	0.087179	0.09534	3
MABM	0.0000243	96.30%	0.034928	0.03626	3
MABM (Fixed λ)	0.0008651	91.80%	0.081862	0.08917	2
EABM	0.0000198	97.30%	0.018103	0.01861	4
EABM (Fixed λ)	0.0000462	93.10%	0.083327	0.08947	3
EDWM	0.0000151	97.90%7	0.015273	0.01560	6
EDWM (Fixed λ (1))	0.0000624	97.60%	0.016503	0.01691	5
EDWM (Fixed λ (1) & λ (2))	0.0000195	97.50%	0.019085	0.01957	4
DWM	0.0000130	98.20%	0.016785	0.01709	5
DWM (Fixed λ (1))	7 0.0000155	97.70%	0.027233	0.02788	4
DWM (Fixed λ (1) & λ (2))	0.0000279	96.20%	0.029073	0.03024	3

The highest mean R^2 can be observed from Model: DWM, while the lowest standard deviation for R^2 was found from Model: EDWM. The lowest coefficient of variance was also recorded in Model: EDWM. A per the above Goodness of fit tool, Model: EDWM can be considered as the optimal model.

Table 2: Summary of the goodness of fit of the models after clustering - Model: EDWM

Cluster	Mean R^2	$\lambda(1)$	$\lambda(2)$	Total RSS	Mean RSS
1	97.59%	4.54596	0.26142	0.000662	0.0000237
2	98.08%	4.58622	0.2655	0.000224	0.0000097
3	97.59%	4.43545	0.27492	0.000719	0.0000225
4	95.39%	4.40159	0.26482	0.000142	0.0000177
5	97.98%	4.51697	0.26391	0.000064	0.0000160
6	97.93%	4.59516	0.5495	0.000191	0.0000159
Model with Clustering	97.58%				0.0000187
Model w/o Clustering	97.54%	4.58622	0.26866	0.002082	0.0000195

It was found that the overall mean R^2 has improved to 97.58% from 97.54% when different parameters are calculated for different clusters under model 4. Total Residual Sum of squares, as well as Mean Residual Sum of squares, also have reduced. Other than cluster 6, the coefficient value for $\lambda(1)$ and $\lambda(2)$ is very close to each other in all the other clusters. Further, the coefficient value for $\lambda(1)$ and $\lambda(2)$ for the model without cluster also much closer to the coefficient values of individual clusters.

Table 3: Summary of the goodness of fit of the models after clustering - Model: DWM

Cluster	Mean R^2	$\lambda(1)$	$\lambda(2)$	Total RSS	Mean RSS
1	97.37%	4.15483	0.04204	0.000742	0.0000265
2	96.24%	4.02857	0.04205	0.000404	0.0000175
3	95.38%	3.92757	0.04206	0.001241	0.0000388
4	94.40%	3.95787	0.04203	0.000185	0.0000232
5	98.72%	3.94575	0.04201	0.000045	0.0000112
6	96.48%	4.24702	0.5495	0.000308	0.0000257
Model with Clustering	96.26%			0.002925	0.0000273
Model w/o Clustering	96.15%	4.11246	0.04201	0.002983	0.0000279

As per Table 3, the overall mean R^2 has improved to 96.26% from 96.15% when different parameters are calculated for different clusters under model 5. Total Residual Sum of squares, as well as Mean Residual Sum of squares, are reduced too. The coefficient value for $\lambda(1)$ is similar for clusters 3,4 and 5 while the other three clusters (i.e., 1,2,6) are close together. But all the coefficient values for $\lambda(2)$ are very close to each other irrespective of clusters. Further, the coefficient values for $\lambda(1)$ of clusters 1,2 and 6 and $\lambda(2)$ for all clusters are much closer to the coefficient values of the model without clustering. It is visible that reconstructing $\lambda(1)$ and $\lambda(2)$ for clustered data set has not improved the predictive power or the accuracy significantly compared with none clustered data set with fixed $\lambda(1)$ and $\lambda(2)$.

CONCLUSION

The entire yield curve was able to represent four latent factors ($\beta0$, $\beta1$, $\beta2$, $\beta3$) and two-scale factors $\lambda1$, $\lambda2$ in model-4 and three latent factors ($\beta0$, $\beta1$, $\beta2$) and two-scale factor $\lambda1$, $\lambda2$. In order to test the said latent features/loadings on variables, it was decided to apply the neural network. Since we use nine different securities in this study, those data points were considered as Input variables and features (parameters) were considered as output variables in the multilayer perceptron neural network model. It was found the average overall relative error is relatively low in training as well as testing data set. This result indicates four parameters of the Model: EDWN and Model: DWN after fixing for $\lambda1$, $\lambda2$, is capable of representing the input data, which is the interest rate from 3 months to 10 years. The Neural Network was performed on the data set with three parameters of Model: DWN after fixing for $\lambda1$, $\lambda2$.

It is important to select data out of the sample (2010 to 2018), to generalize the applicability of the proposed model. It was found that Model:EDWN able to estimate the future interest rate movements with an R^2 value greater than 98%, corresponding R^2 for Model: DWN was only above 91%. Finally, it was found Model: EDWN is better than Model-5 when it is used out of sample data with a larger number of securities (training data set included only yields of nine different securities, whereas test data set - out of sample included yields of forty-two (42) different securities) which has charactoristics of Dobbie and Wilkie (1977) model.

Many researchers have carried out yield curve modeling by fixing λi for convenience, simplicity, and numerical trustworthiness (Diebold et al., 2006). During this research, both approaches were considered and models were compared after fixing λi and not fixing λi. As per Diebold et al. (2006) who brought the economical interpreted the NS Model state λi as the one which determines the maturity at which the loading on the medium-term or curvature factor achieves its maximum.

LIMITATIONS AND FUTURE RESERACH DIRECTIONS

Most of the studies under Term Structure Interest rates have focused on the forward yield curve or spot yield curve (Zero-coupon bond curve). But in this study, the main focus was on yield to the maturity yield curve. Therefore, it would be interesting to construct the yield curve model for "forward yields" as well as "zero-coupon bond yields".

Sri Lanka does not have longer-term securities with high liquidity in a decade ago. The yields were available for 20 and 30 years only after 2014. It was not sufficient to have four-year data for the study. Therefore, the time to come better estimation can be derived from extended yield curve (at least yield curve up to twenty years). Further, the available tenors of securities were also limited. After 5 years the available (active) bonds were six years and 10 years. Eight years were available only after 2014. It would have been better to obtain yields for the securities that have maturity tenor of 7, 8 and 9 years for this study. Going forward, an optimal model can be rebuild using more than nine securities as the accurate traded data available.

One other limitation was unable to observe invert yield curve data for the considered period for Sri Lanka Government Bond yields. Hence, it was not possible to test the model for the inverted pattern of the yield curve.

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